Multiquark hadrons in a quark model

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Exotic states

- ‘Pentaquarks’ ($q^4\bar{q}$)
  - Θ^+, Ξ, …
  - Baryon resonances with a large width

    → $\Lambda(1405)$, $\Delta(1232)$, $N(1440)$, ...

    S.T. and K Shimizu, P.R. **C76**, 035204(07);

    S.T. and K Shimizu, P.R. **C79**, 045204(09)

Adding ($q\bar{q}$) is important because of the parity.

A baryon with the meson cloud can be understood as a ‘pentaquark’.

Non $q\bar{q}$ meson candidates

- $(q\bar{q})^2$ Mesons?

Refs. Particle Data Group
C. Amsler et al., Phys. Lett., B667, 1 (08)

  - $qc\bar{q}\bar{c}$?

    $X(3872)$, $Y(4140)$, $Z(4430)^{\pm}$, ...

  - $qs\bar{q}\bar{c}$?

    $D^*_{s0}(2317)^{\pm}$, $D^*_{s1}(2460)^{\pm}$,

    $D^*_{s1}(2536)^{\pm}$, $D^*_{s2}(2573)^{\pm}$

  - $qs\bar{q}\bar{s}$ or K^+K^-?

    $a_0(980)$, $f_0(980)$, $X(1576)$

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X(3872) facts

- X(3872)
  - $M(X) = 3872.3 \pm 0.8$ MeV
  - $\Gamma = 3.0+2.1-1.7$ MeV
  - $I^G(J^{PC}) = 0^?(?^{?+})$ $J^{PC} = 1^{++}, 2^{-+}$?
  - $I=0 \leftarrow$ No $X^\pm$ found
  - In $B^\pm \rightarrow K^\pm X$, $p\bar{p} \rightarrow X$, but not in e^+e^-
  - decay mode $X \rightarrow J/\psi\, \pi^2$, $J/\psi\, \pi^3$, $J/\psi\, \gamma$
  - $\Gamma(X \rightarrow J/\psi\, \gamma)/\Gamma(X \rightarrow J/\psi\, \pi^2) = 0.14 \pm 0.05$
  - $r(X \rightarrow J/\psi\, \pi^3)/Br(X \rightarrow J/\psi\, \pi^2) = 1.0 \pm 0.4 \pm 0.3$

Abe etal arXiv:hep-ex/0505037v1

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  - I=0 ← No $X^\pm$ found
  - In $B^\pm \to K^\pm X$, $p\bar{p} \to X$, but no

Isospin sym. breaking?

  - decay mode $X \to J/\psi\, \pi^2$, $J/\psi\, \pi^3$, $J/\psi\, \gamma$
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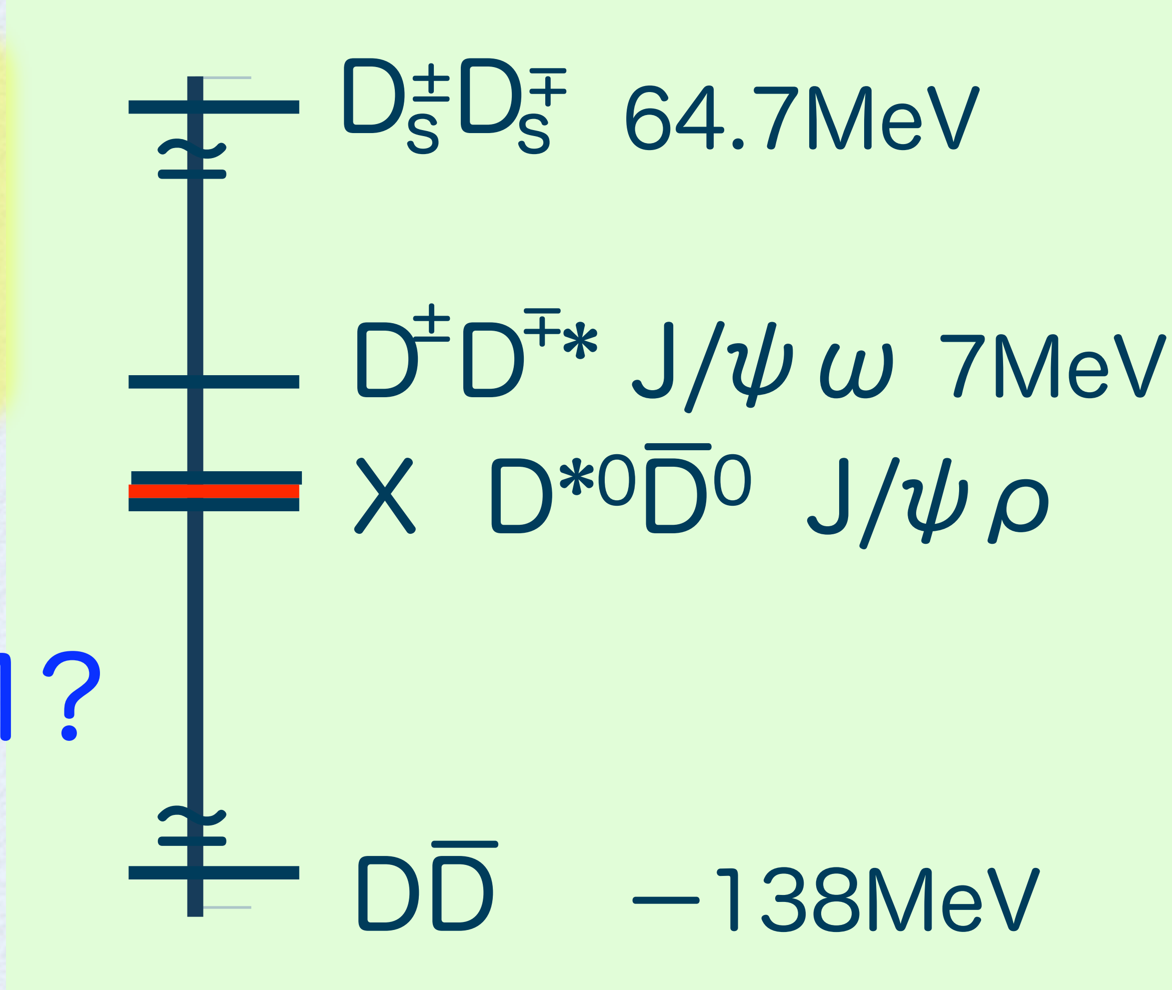
X(3872) facts

- X(3872) $(uc\bar{u}\bar{c},\ dc\bar{d}\bar{c})$ threshold
  - $J/\psi\,\omega$ = 3879.6MeV
  - $D^{\pm}D^{*\mp}$ = 3879.9MeV
  - $J/\psi\,\rho$ = 3872.4MeV
  - $D^{0}\bar{D}^{*0}$ = 3871.8MeV

M(X) = 3872.3±0.8 MeV



Isospin is violated.

$u\bar{u}c\bar{c}$ rather than I=0,1?

e.g. N. Tornqvist B590, 209 (04)

X(3872) facts

- ‘charged $X^{\pm}$ $(uc\bar{d}\bar{c})$’ threshold
  - $D^{\pm}D^{*0}$ = 3876.6MeV
  - $D^{0}D^{*\mp}$ = 3875.1MeV
  - $J/\psi\ \rho$ = 3872.4MeV

    ・・・
  - $J/\psi\ \pi^{+}$ = 3236.5MeV

$M(X^0)$
= 3872.3±0.8 MeV

If there is no special symmetry, there should not be a bound state. Even if so, since ρ-width is large (∼150MeV), it would be difficult to make a clear peak.

Realistic Calc. - $q\bar{q}c\bar{c}$

- Resonating group method approach

$$
\begin{aligned}
\Psi &= \sum c_{km} \psi^c_m \psi^f \psi^\sigma \psi^{orb}_k \\
\psi^f &= u\,c\,\overline{u}\,\overline{c},\ d\,c\,\overline{d}\,\overline{c}\ \text{ for the neutral; } u\,c\,\overline{d}\,\overline{c} \text{ for the charged} \\
\psi^c &= (\psi^c(1)\psi^c(4))(\psi^c(2)\psi^c(3)),\ (\psi^c(1)\lambda\psi^c(4))(\psi^c(2)\lambda\psi^c(3)) \\
\psi^\sigma &= J/\psi \ \text{ is spin 1} \\
\psi^{orb} &= \sum c_k \phi_1(\text{meson})\phi_2(\text{meson})\chi_k(\text{relative}) \\
&= (\overline{D}^0 D^{*0} - \overline{D}^{*0} D^0)\chi,\ (D^+ D^{*-} - D^{*+} D^-)\chi, \\
&\quad (J/\psi\, \rho^0)\chi,\ (J/\psi\, \omega)\chi \ \text{ for the neutral} \\
&= (\overline{D}^0 D^{*+})\chi,\ (D^+ \overline{D}^{*0})\chi,\ (J/\psi\, \rho^+)\chi \ \text{ for the charged}
\end{aligned}
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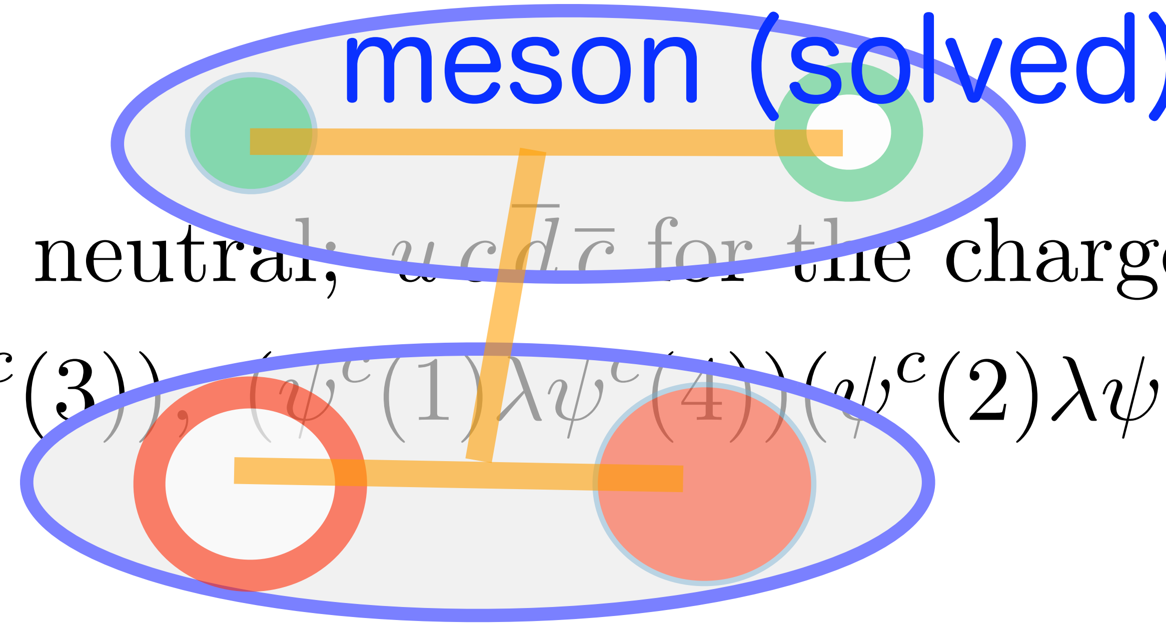
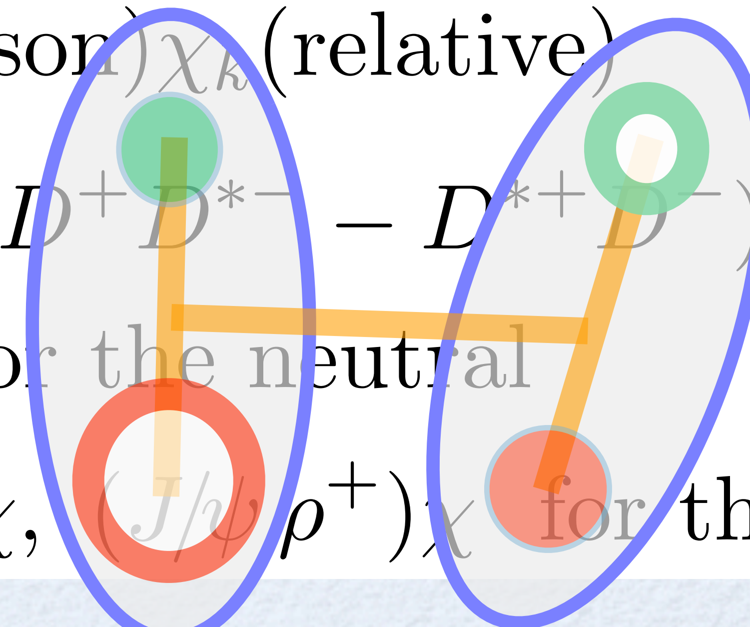
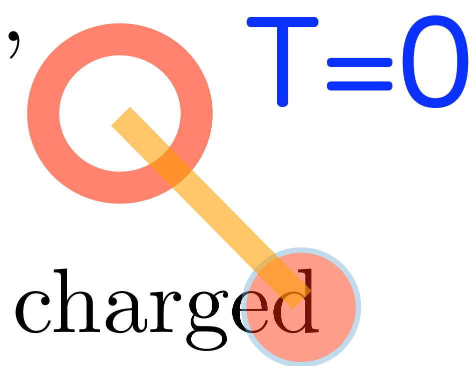


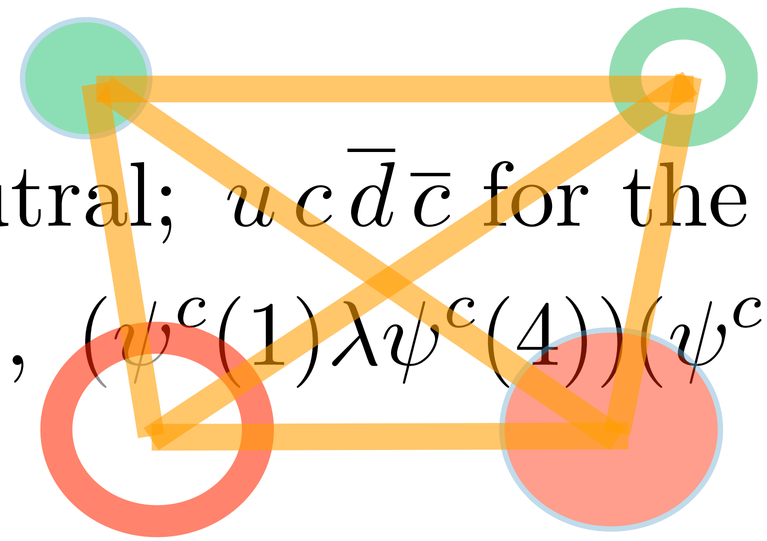




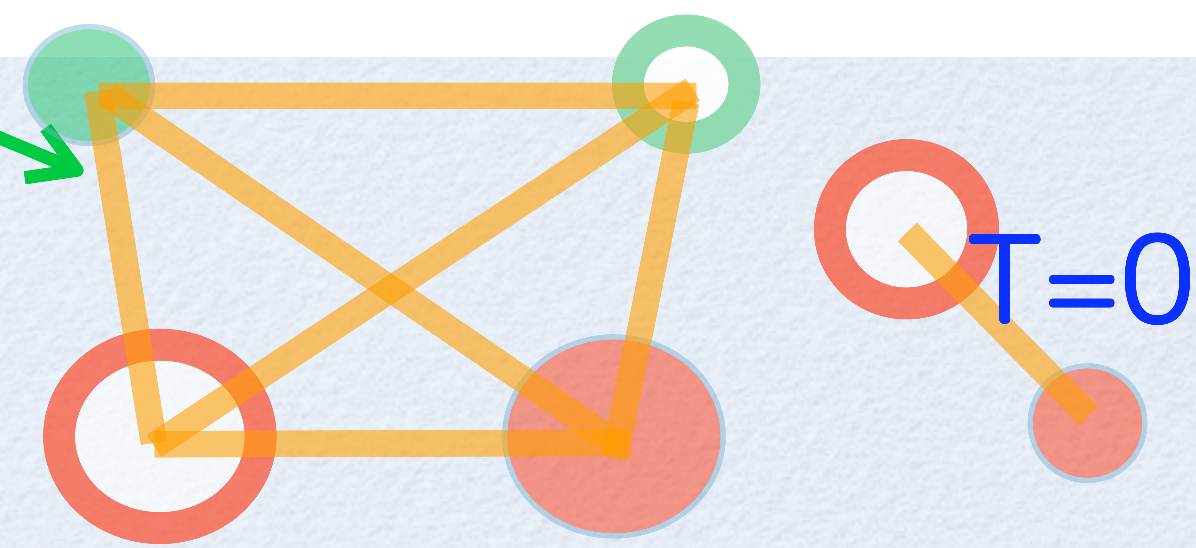
Realistic Calc. - $q\bar{q}c\bar{c}$

- short-range deformation

$$
\begin{aligned}
\Psi &= \sum c_{km}\psi^c_m \psi^f \psi^\sigma \psi^{orb}_k \\
\psi^f &= u\,c\,\bar{u}\,\bar{c},\ d\,c\,\bar{d}\,\bar{c} \text{ for the neutral; } u\,c\,\bar{d}\,\bar{c} \text{ for the charged} \\
\psi^c &= (\psi^c(1)\psi^c(4))(\psi^c(2)\psi^c(3)),\ (\psi^c(1)\lambda\psi^c(4))(\psi^c(2)\lambda\psi^c(3)) \\
\psi^\sigma &= J/\psi \ \text{ is spin 1} \\
\psi^{orb} &= \sum c_k \exp[-\sum \beta^{(k)}_{ij} r^2_{ij}]
\end{aligned}
$$



'all' the orbital correlations



Hamiltonian for quarks

- H = Nonrela Kin + linear Conf
  + OGE + Ins + π, σ exch + ele-mag
- Kinetic term with u, d mass difference

$$K = \sum m_i + \frac{p_i^2}{2m_i^2} \quad \text{for c-quark}$$

D⁰-D± mass diff etc.
is reproduced.

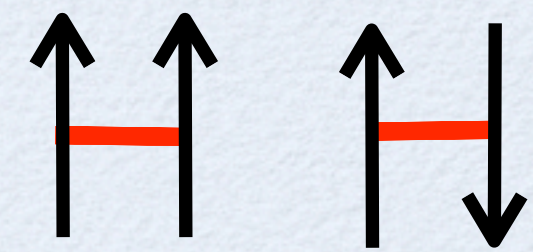
$$K = \sum \overline{m} + \frac{p_i^2}{2\overline{m}^2} + \Delta m_i \quad \text{for u-, d-quarks}$$

where

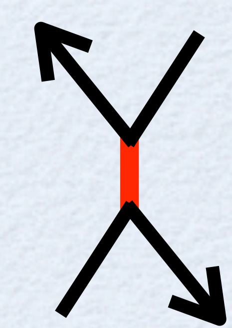
$$\overline{m} = (m_u + m_d)/2 \quad \text{with} \quad \Delta m_i = \begin{cases} \sqrt{m_i^2 + p_i^2} - \sqrt{\overline{m}^2 + p_i^2} \\ 0 \quad \text{for color}_8 \text{ quark pairs} \end{cases}$$

Hamiltonian for quarks

- H = Nonrela Kin + linear Conf
  + OGE + Ins + π, σ exch
  - OGE



$$V_{\rm ele} = \sum_{i<j} -\pi\alpha_s \frac{\lambda_i\cdot\lambda_j}{4}\frac{1}{2}\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}\right)\delta^3(\boldsymbol{r}_{ij})$$



$$V_{\rm CMI} = \sum_{i<j} -\pi\alpha_s \frac{\lambda_i\cdot\lambda_j}{4}\sigma_i\cdot\sigma_j\frac{2\xi_i\xi_j}{3m_u^2}\delta^3(\boldsymbol{r}_{ij})$$

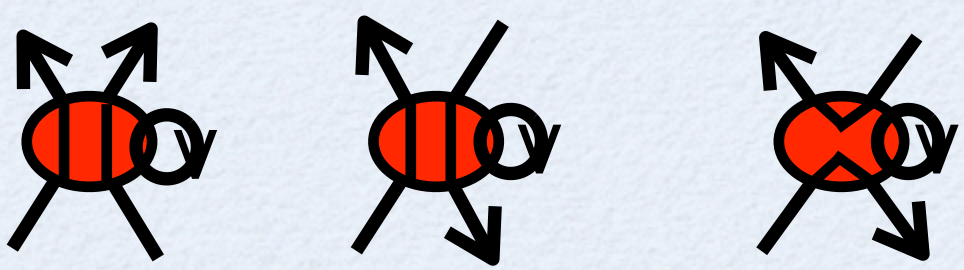
$$V^{(a)}_{\rm OGE} = \sum_{i<j}\frac{1}{24}\left(\frac{16}{3}+\lambda_i\cdot\lambda_j\right)(3+\sigma_i\cdot\sigma_j)\mathcal{P}_{ij}\ \pi\alpha_s\frac{1}{4mm'}\delta^3(\boldsymbol{r}_{ij})$$

Hamiltonian for quarks

- Ins (affects only light quark pairs.)

$$V_{\mathrm{INS}} = \sum_{i<j} \frac{V_0}{2} \xi_i \xi_j \left(1 + \kappa \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \underline{\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j}\right) \mathcal{P}'_{ij} \delta^3(\boldsymbol{r}_{ij})$$

$$V_{\mathrm{INS}}^{(a)} = \sum_{i<j} -\frac{V_0}{2} \xi_i \xi_j \mathcal{P} \mathcal{P}'' \left(1 - \frac{3}{32} \lambda_i \cdot \lambda_j + \frac{9}{32} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j\right) \delta^3(\boldsymbol{r}_{ij})$$



Estimate by $(0s)^4$

- Effects of the interaction on $q\bar{q}$ pairs

  Rough sizes are obtained from N Δ, and η'-η mass differences.

Color	Spin	Flavor	CMI	OgE-a	Ins	E[MeV]	States
1	0	1	-16	0	12	84	η_1
1	0	8	-16	0	-6	-327	π η_8
1	1	1	16/3	0	0	63	ω
1	1	8	16/3	0	0	63	ρ
8	0	1	2	0	3/4	41	
8	0	8	2	0	-3/8	15	
8	1	1	-2/3	9/2	9/4	97	DD* attraction
8	1	8	-2/3	0	-9/8	-34	

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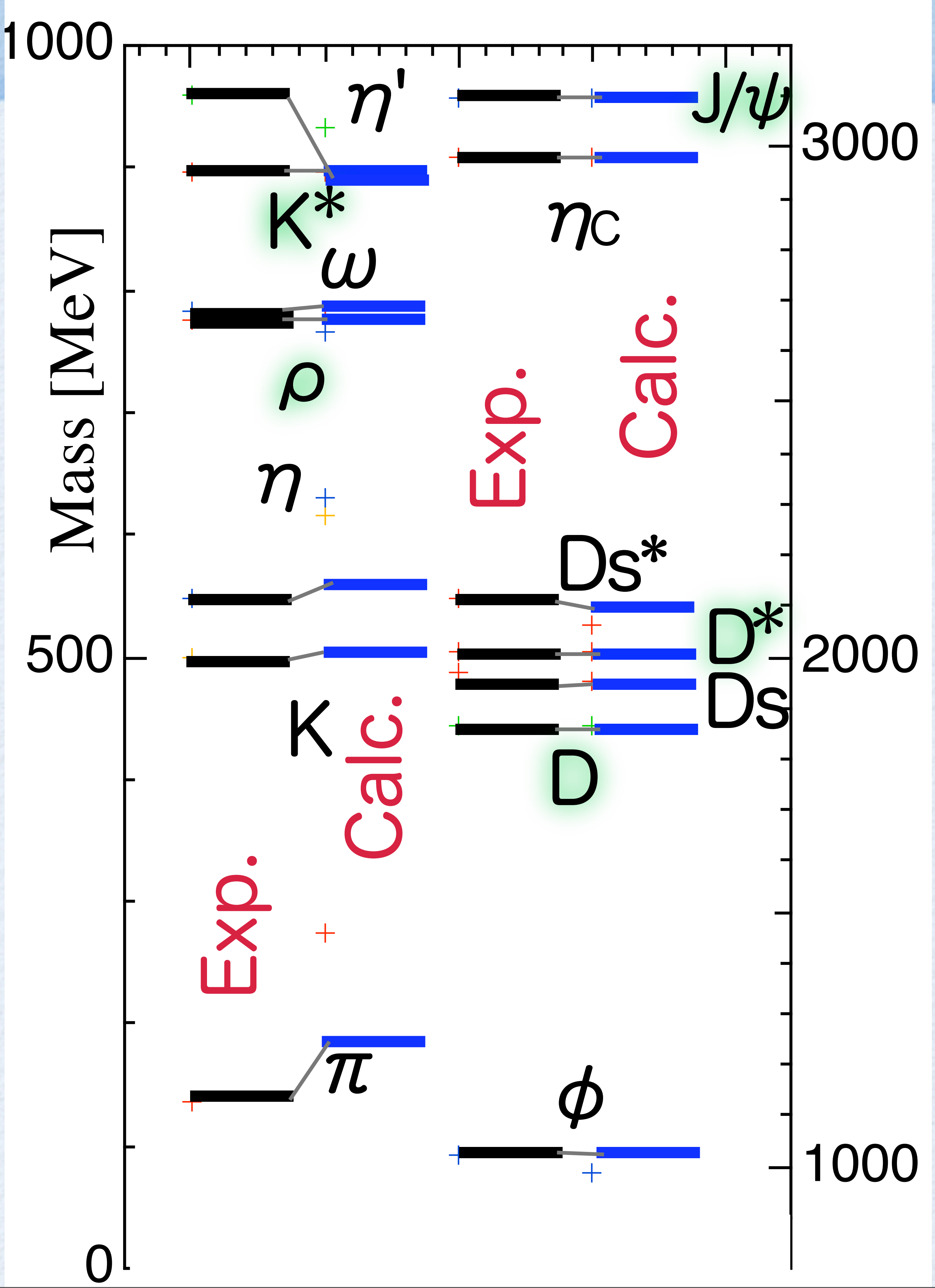
8	1	8	-2/3	0	-9/8	-34	DD* attraction

Realistic Calc. - mesons

m_u = 311.5 MeV, m_d = 314.5 MeV
m_c = 1250 MeV

a_{conf} = 172.4 MeV/fm
α_s = 0.58
$V_{0,ins}$ = -113 MeV/fm^3 (p_{III}=0.4)
ξ_{cu}=0.54
ξ_{cc}=0.288
Λ_g = 3.3 fm^{-1}
$g_8^2/4\pi$=0.69
m_σ=800 MeV
Λ_σ= 8.9fm^{-1}
Λ_π= 1.5fm^{-1}



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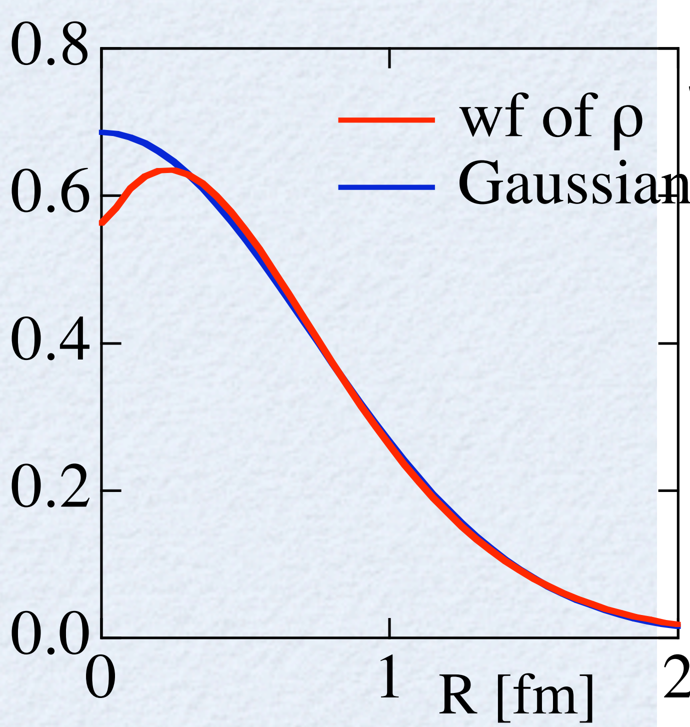
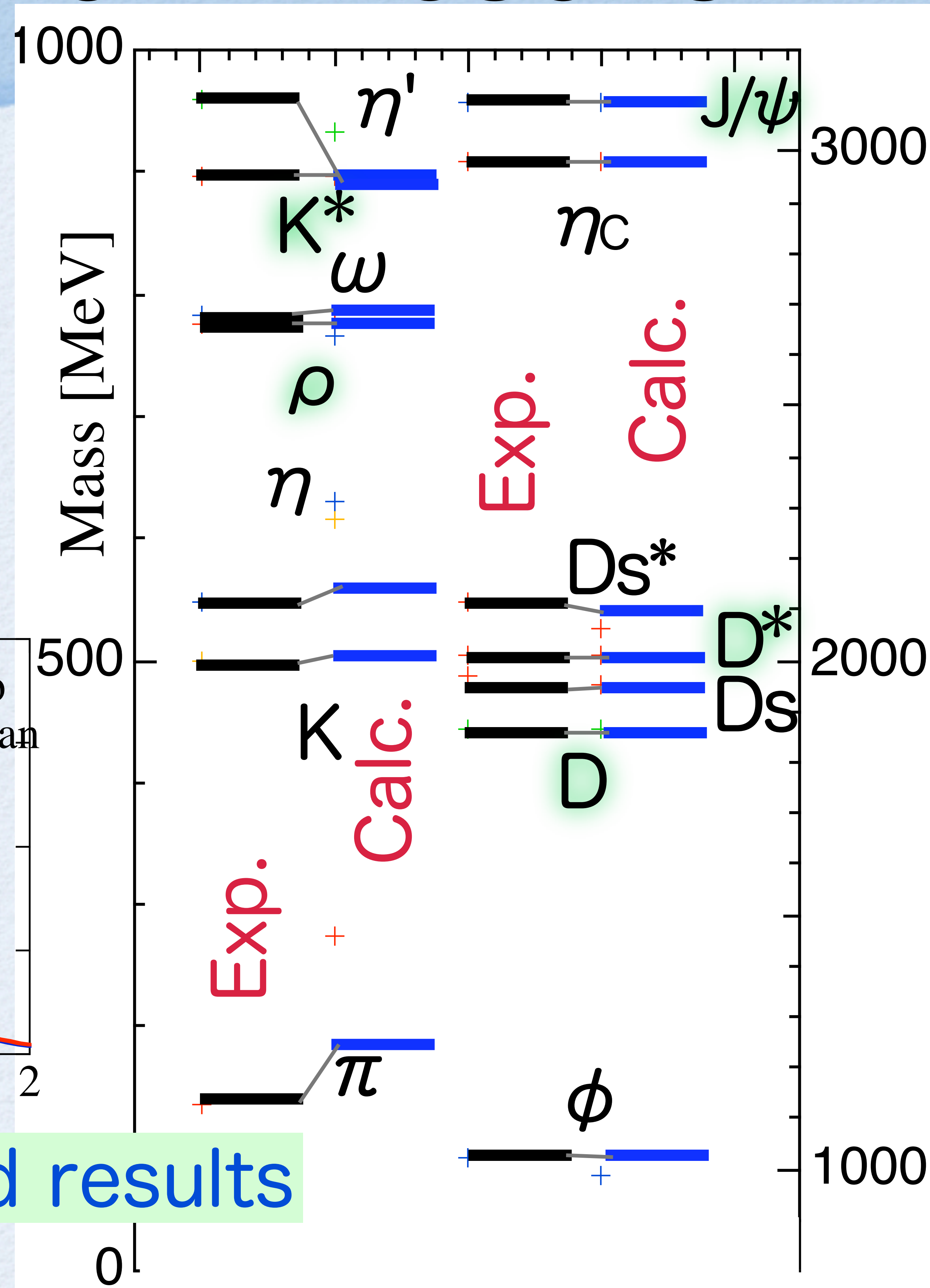




Solved results

Realistic Calc. - mesons

- meson masses

	calc.	exp.		calc.	exp.
ρ^0	775.50	775.50	D^0D^{*0}	3871.20	3871.20
ω	782.65	782.65	$J/\psi\,\rho^0$	3872.42	3872.42
J/ψ	3096.92	3096.92	$D^{\pm}D^{*\pm}$	3879.59	3879.30
D^0	1864.56	1864.50	$J/\psi\,\omega$	3879.57	3879.57
D^{*0}	2006.64	2006.70			
$D^{\pm}$	1869.54	1869.30			
$D^{*\pm}$	2010.05	2010.00			

Coupling to $c\bar{c}$

- $q(q\bar{q})\to q$, $\bar{q}(q\bar{q})\to\bar{q}q$ transfer interaction

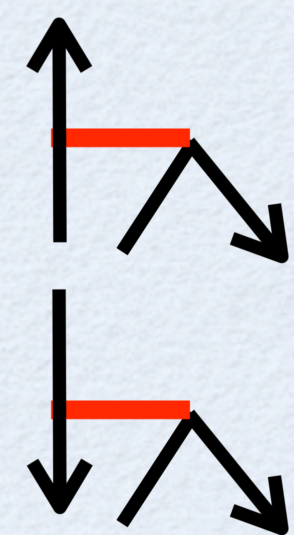
$$V_{i;j\bar{k}} = \lambda_i \cdot \lambda_{\bar{k}j} \frac{\alpha_s}{4} \frac{\pi}{m_a^2} \left[\left(\frac{\boldsymbol{k}}{2m_a} - \frac{\boldsymbol{p}_i + \boldsymbol{p}_i' + i\boldsymbol{\sigma}_i \times \boldsymbol{k}}{2m_i} \right) \cdot \boldsymbol{\sigma}_{\bar{k}j} \right] \delta^f_{\bar{k}j}$$

- consider only btw $(0s)^4$ and $(1p)$

$V_{tr} = |(q\bar{q})^2(0s)^4\rangle V_{OGE} \langle q^2(1p)|$

Pole energy = 3950MeV

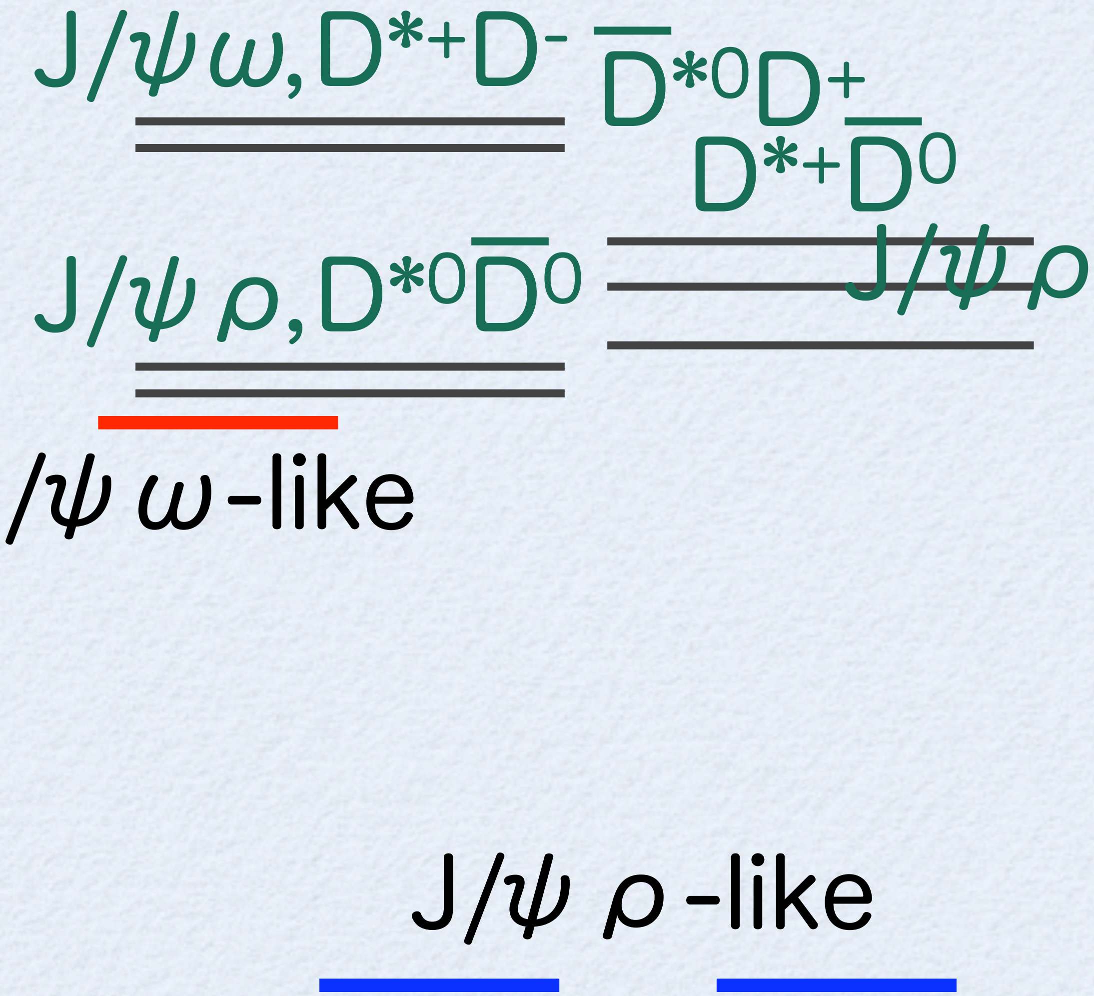
Godfrey Isgur PRD32, 189 (85)



Isospin 0 v.s. 1

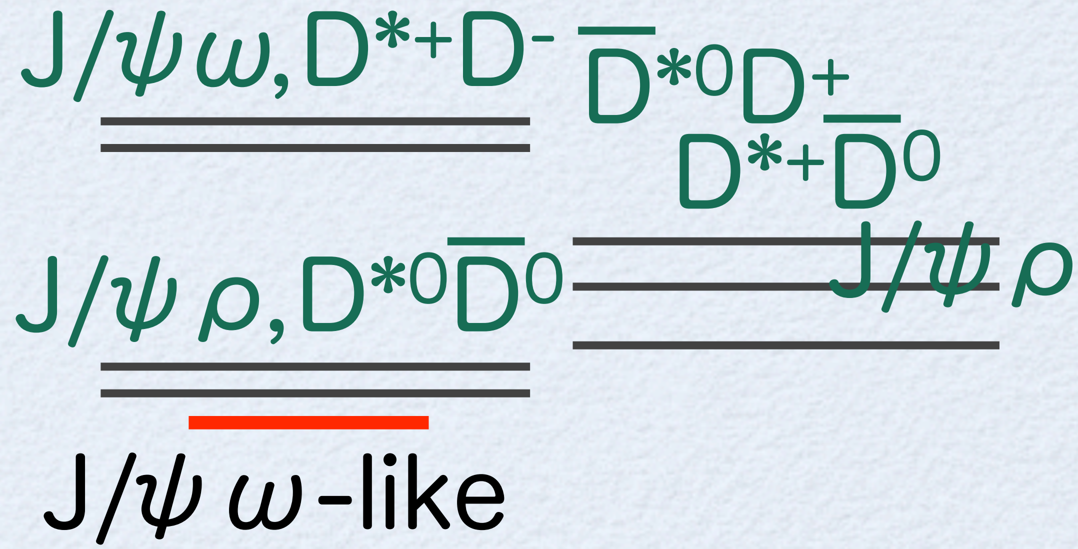
- Case 1

  Small $c\bar{c}$ compo.
  Deeply bound I=1

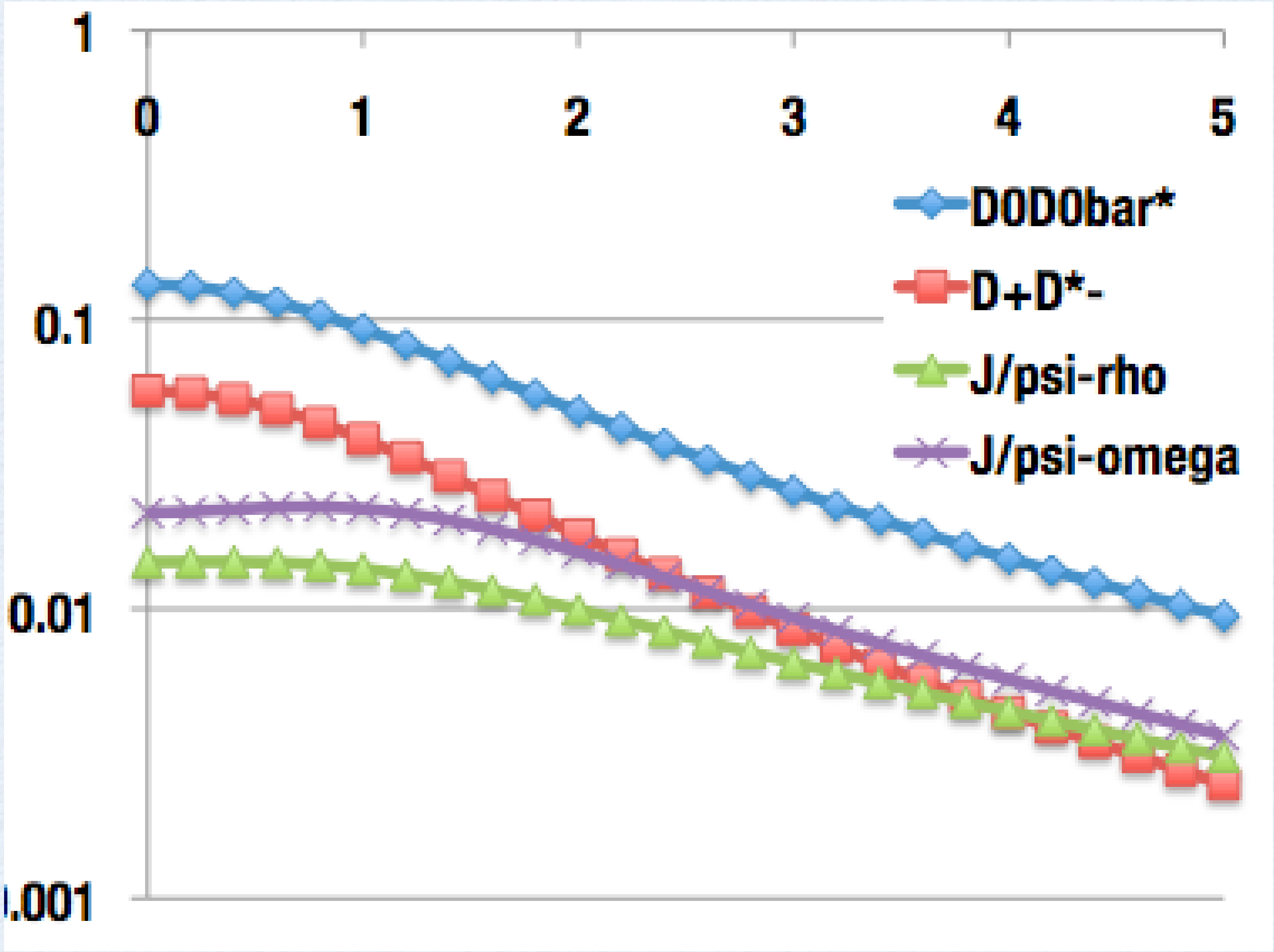


- Case 2

  Large $c\bar{c}$ compo.
  No bound I=1



wave function (case 2)



$(q\bar{q}c\bar{c}+c\bar{c})$ Case-1

- Stronger attraction for $qc\bar{q}\bar{c}$, weaker coupling to $c\bar{c}$.

$J/\psi\,\omega$ $J/\psi\,\rho$ DD^*

state	BE	C_1T0	C_1T1	C_8T0	C_8T1	pole
$J/\psi\,\rho^0$-like	21.6	0.01	0.26	0.03	0.67	0.00
$J/\psi\,\omega$-like	0.4	0.48	0.01	0.43	0.06	0.01
$J/\psi\,\rho^+$-like	20.5	-	0.30	-	0.69	-

1 : 25

13 : 1

×(phase space) → $Br(X\to J/\psi\,\pi^3)/Br(X\to J/\psi\,\pi^2)$

$(q\bar{q}c\bar{c}+c\bar{c})$ Case-2

- Weaker attraction for $qc\bar{q}\bar{c}$, stronger coupling to $c\bar{c}$.

		$J/\psi\,\omega$	$J/\psi\,\rho$		DD*	
state	BE	C_1T0	C_1T1	C_8T0	C_8T1	pole
$J/\psi\ \rho^0$-like	-					
$J/\psi\ \omega$-like	5.2	0.11	0.04	0.44	0.09	0.37
$J/\psi\ \rho^+$-like	-					

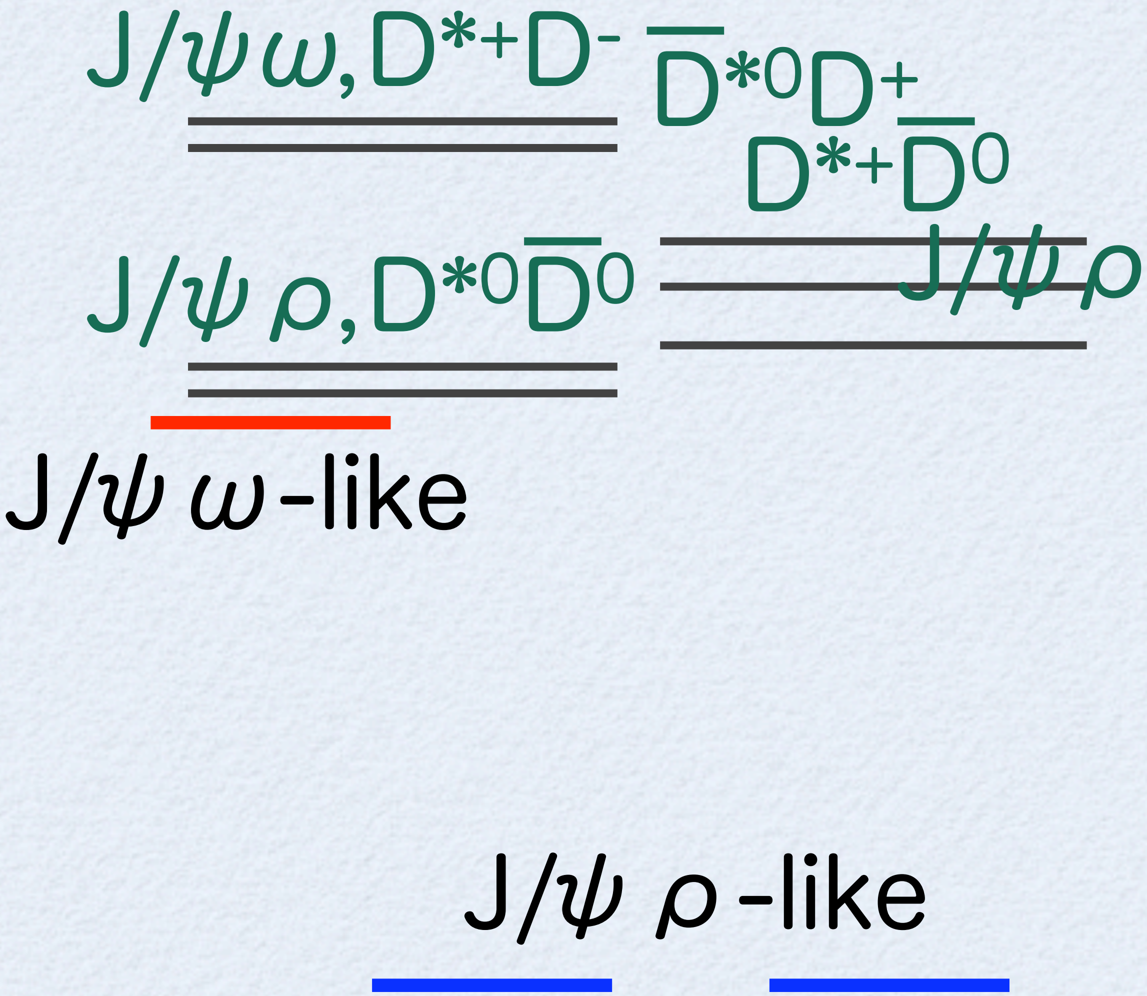
4 : 1

×(phase space) → $Br(X \to J/\psi\,\pi^3)/Br(X \to J/\psi\,\pi^2)$

Isospin 0 v.s. 1

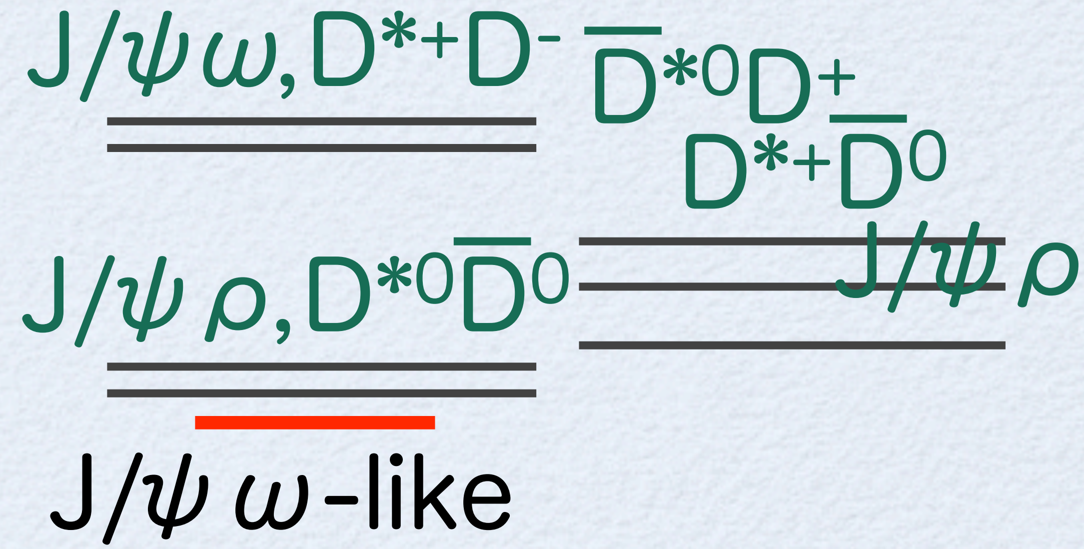
- Case 1

  Small $c\bar{c}$ compo.
  Deeply bound I=1



- Case 2

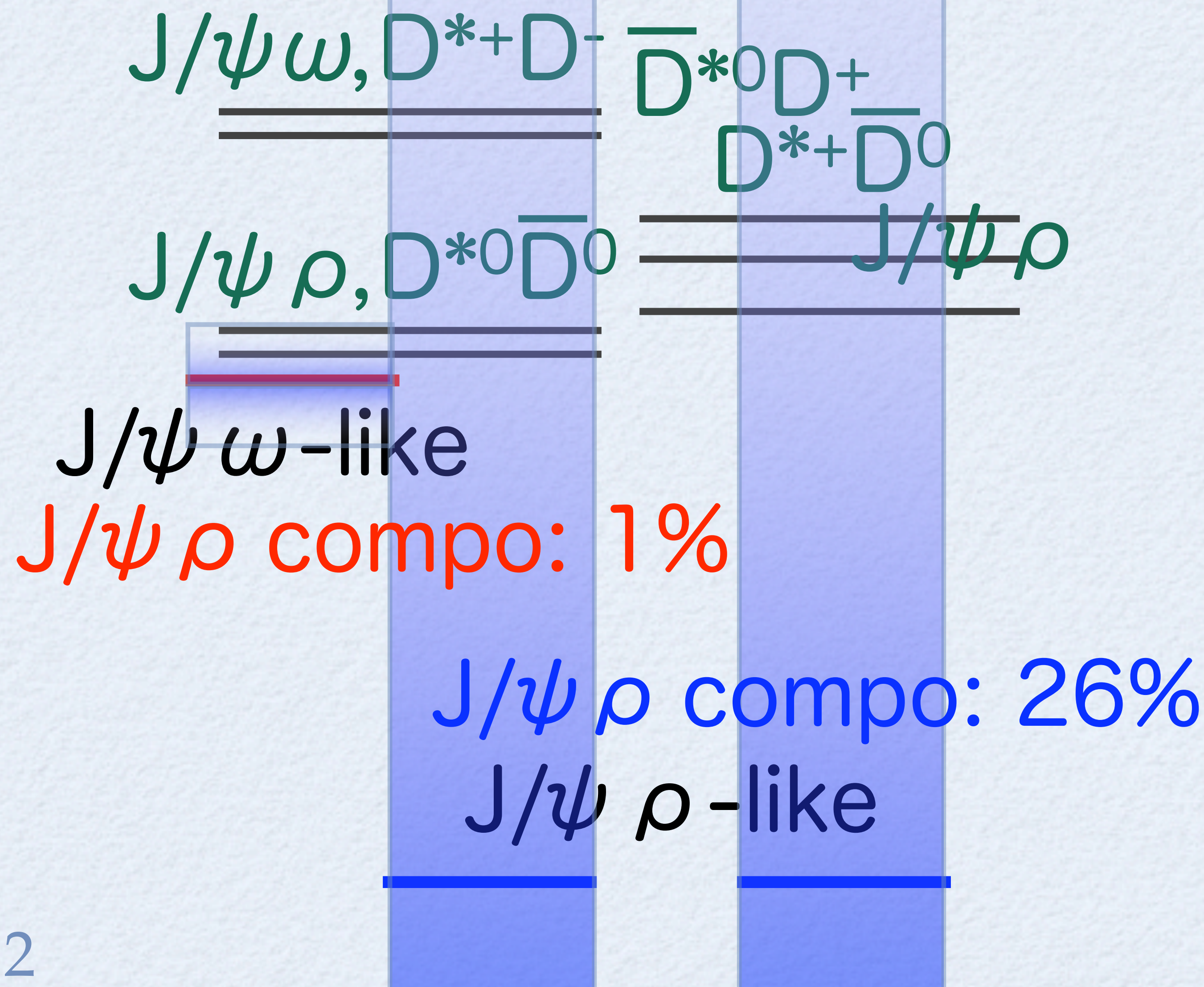
  Large $c\bar{c}$ compo.
  No bound I=1



Isospin 0 v.s. 1

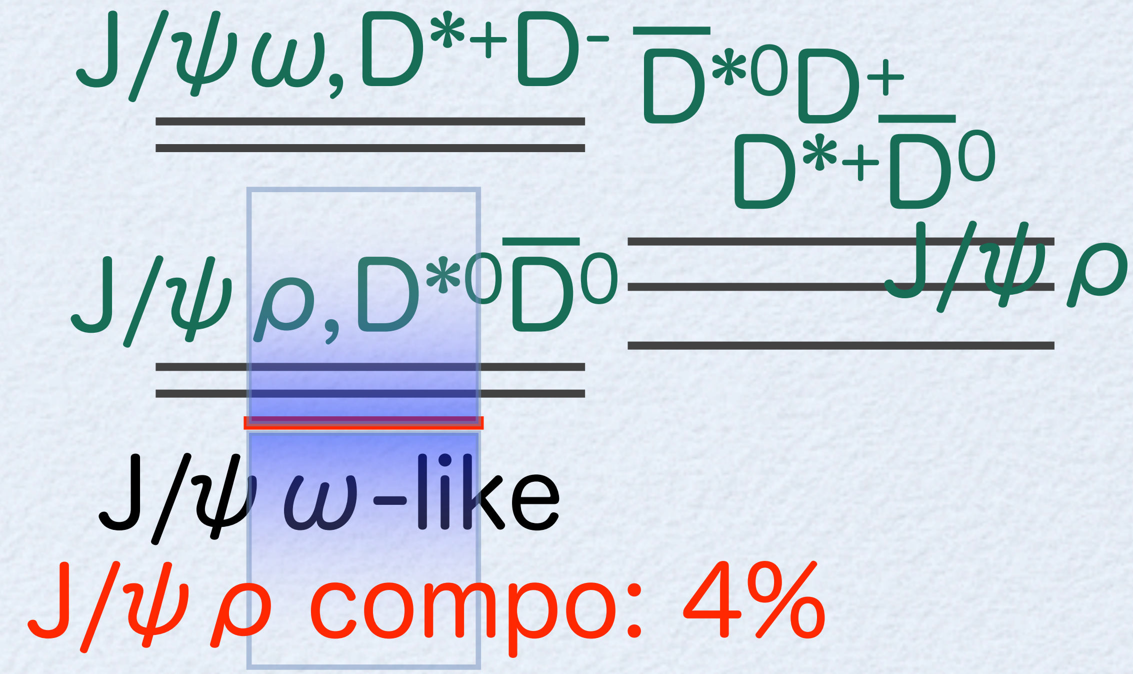
- Case 1

  Small $c\bar{c}$ compo.
  Deeply bound I=1



- Case 2

  Large $c\bar{c}$ compo.
  No bound I=1



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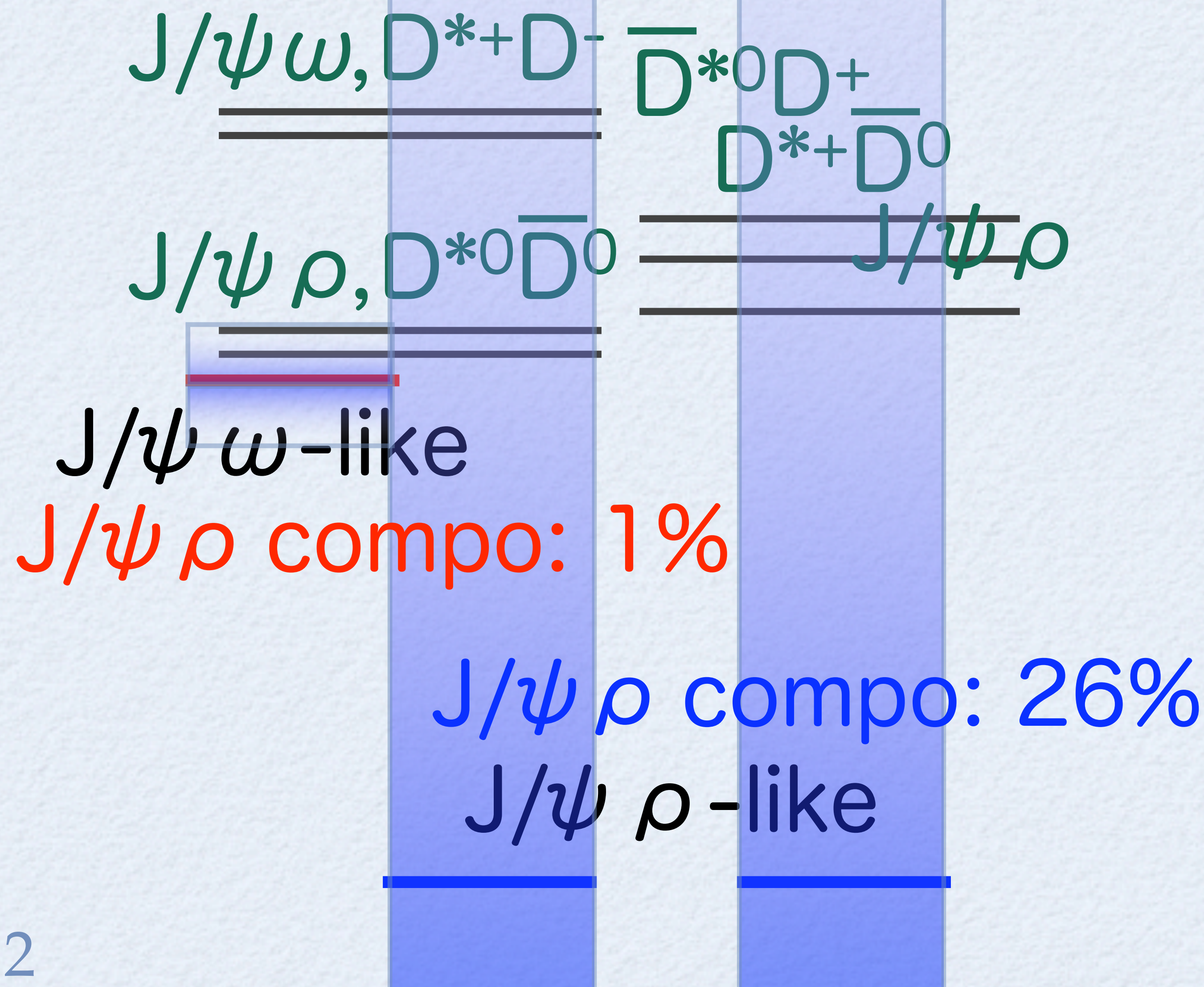

- Case 2

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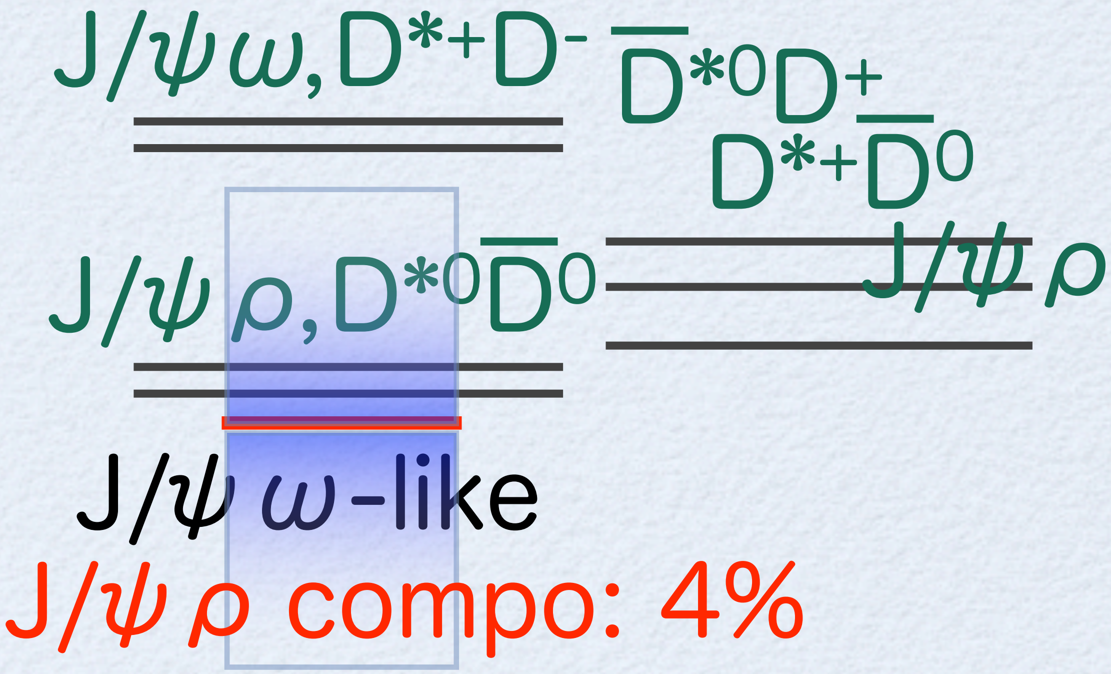
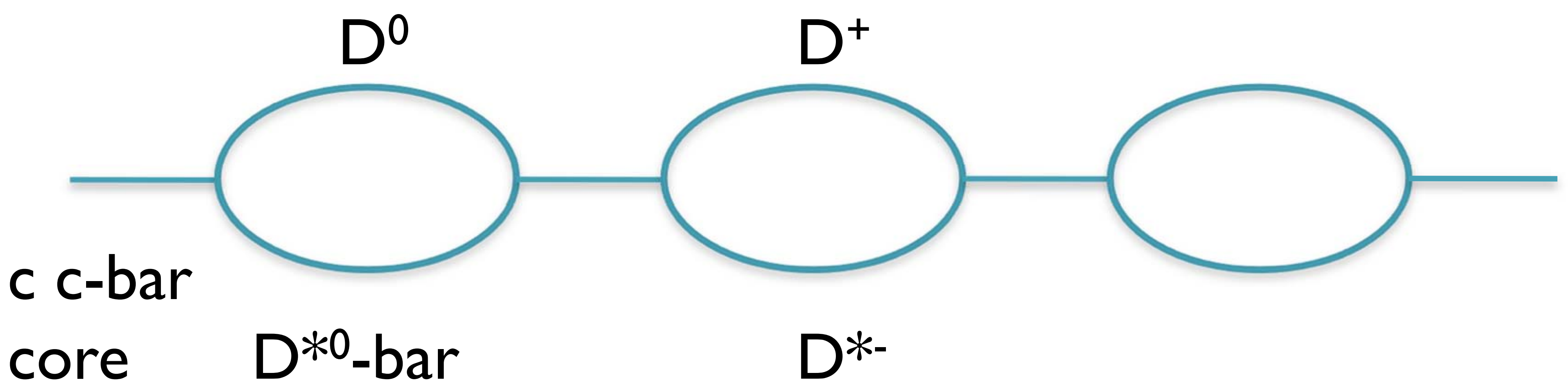


Energy spectrum
→ Takizawa

Coupling between C C-bar core and D^0 D^{*0}-bar, D^+ D^{*-}



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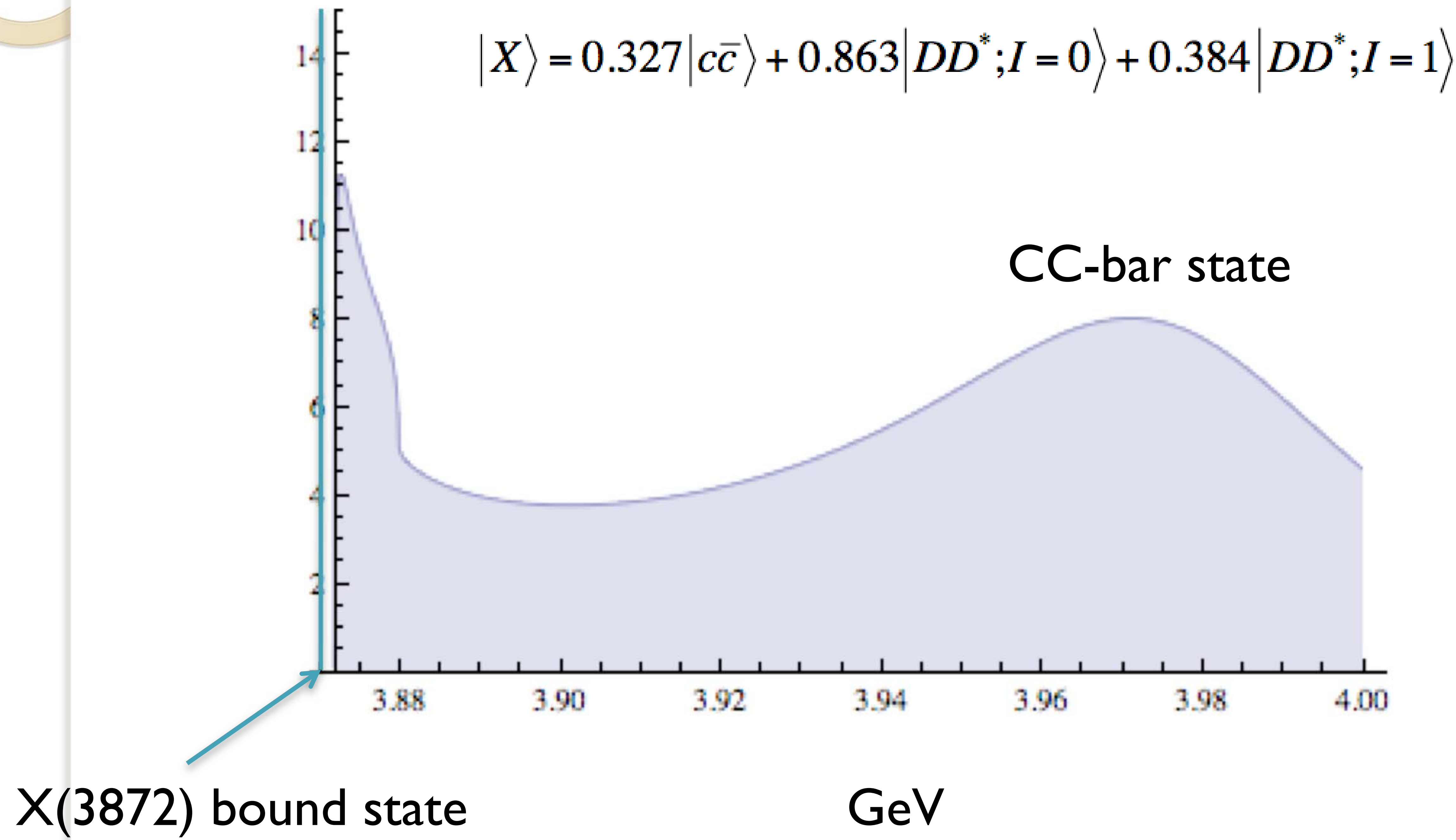
Numerical results: Energy spectrum

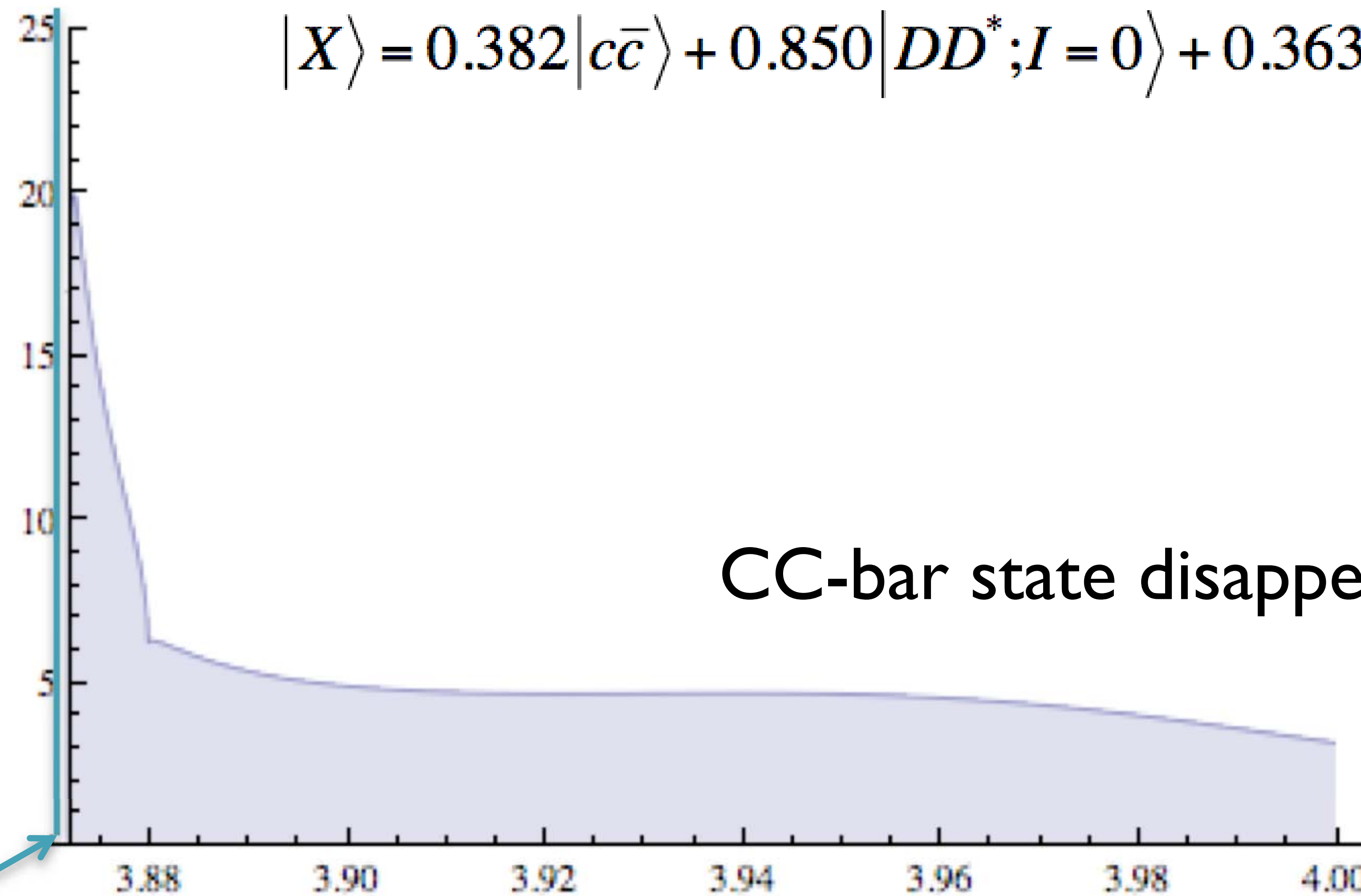
- Lambda = 0.3 GeV



Numerical results: Energy spectrum

- Lambda = 0.5 GeV

$$|X\rangle = 0.382|c\bar{c}\rangle + 0.850|DD^*; I=0\rangle + 0.363|DD^*; I=1\rangle$$



CC-bar state disappears

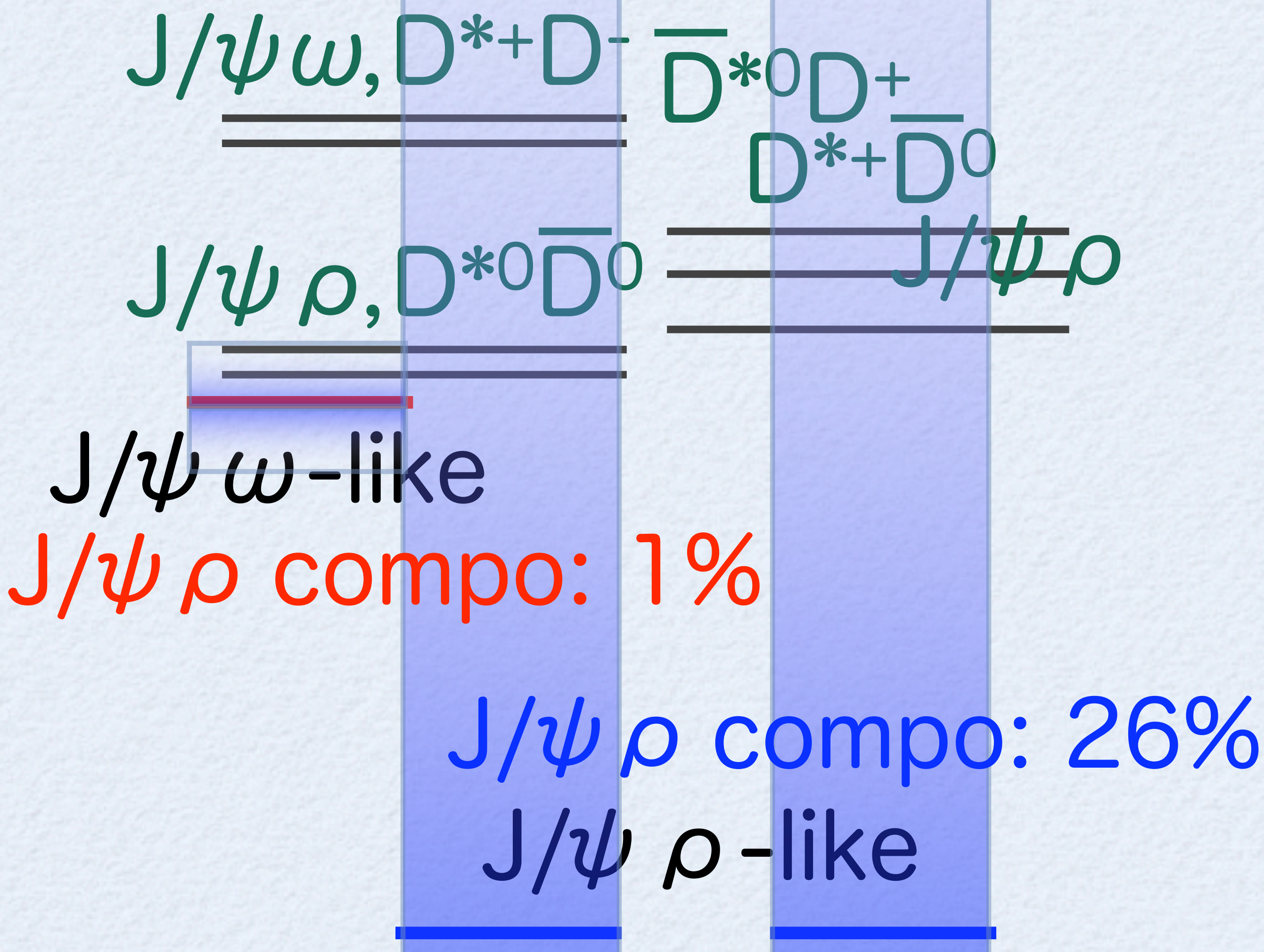
X(3872) bound state

GeV

Isospin 0 v.s. 1

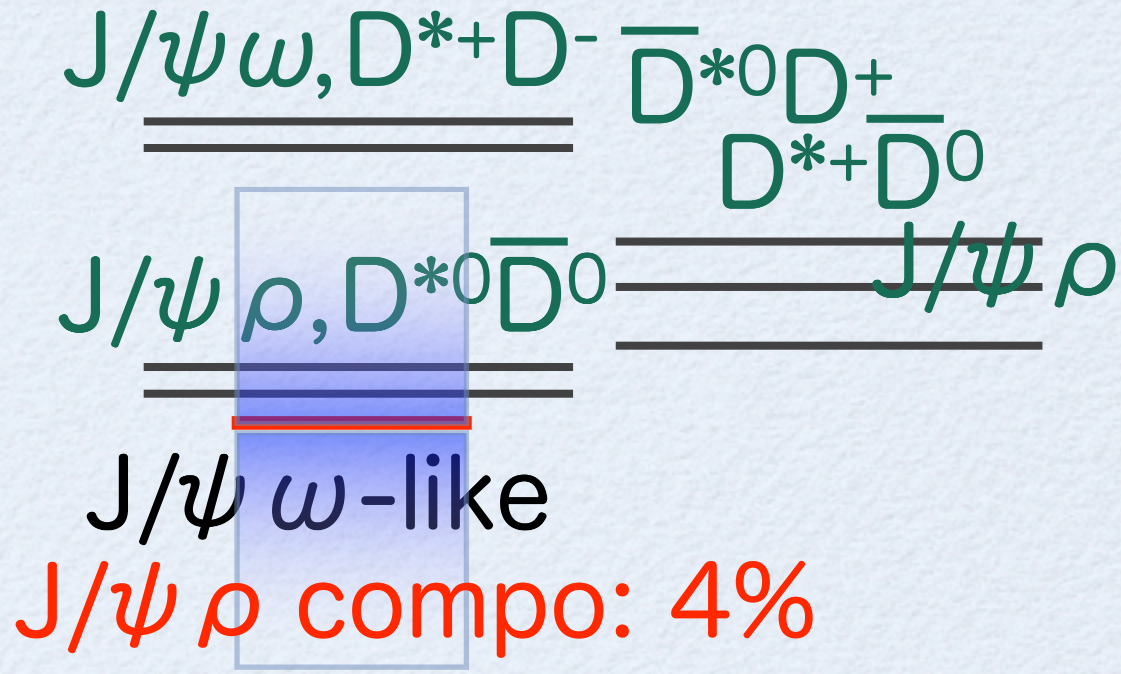
- Case 1

  Small $c\bar{c}$ compo.
  Deeply bound I=1



- Case 2

  Large $c\bar{c}$ compo.
  No bound I=1



Energy spectrum
→ Takizawa

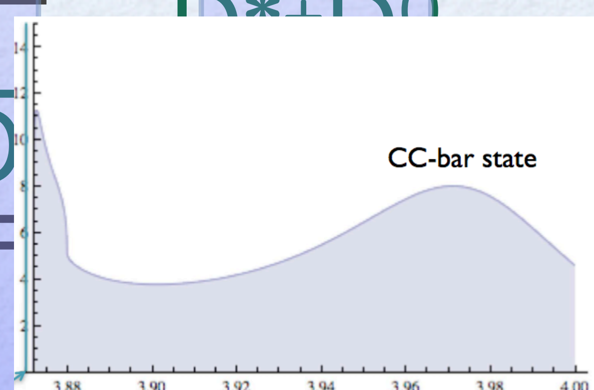
Isospin 0 v.s. 1

- Case 1

Small $c\bar{c}$ compo.
Deeply bound I=1

$J/\psi\,\omega$, $D^{*+}D^-$ $\bar{D}^{*0}D^+$

$J/\psi\,\rho$, $D^{*0}\bar{D}$



$J/\psi\,\omega$-like

$J/\psi\,\rho$ compo: 1%

$J/\psi\,\rho$ compo: 26%

$J/\psi\,\rho$-like

- Case 2

Large $c\bar{c}$ compo.
No bound I=1

$J/\psi\,\omega$, $D^{*+}D^-$ $\bar{D}^{*0}D^+$

$D^{*+}\bar{D}^0$

$J/\psi\,\rho$

$J/\psi\,\rho$, $D^{*0}\bar{D}^0$

$J/\psi\,\omega$-like

$J/\psi\,\rho$ compo: 4%

Energy spectrum
→ Takizawa

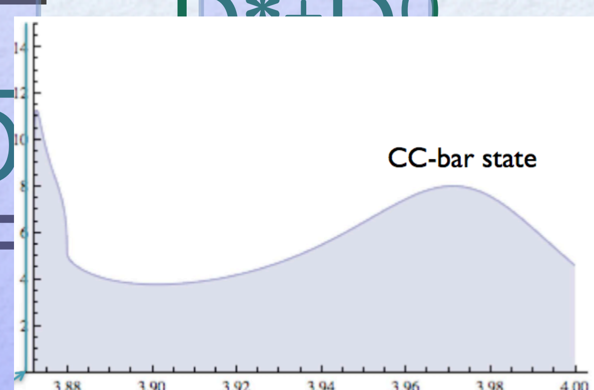
Isospin 0 v.s. 1

- Case 1

  Small $c\bar{c}$ compo.
  Deeply bound I=1

$J/\psi\,\omega$, $D^{*+}D^{-}$ $\bar{D}^{*0}D^{+}$ $D^{*+}\bar{D}^{0}$

$J/\psi\,\rho$, $D^{*0}\bar{D}^{0}$



$J/\psi\,\omega$-like

$J/\psi\,\rho$ compo: 1%

$J/\psi\,\rho$ compo: 26%

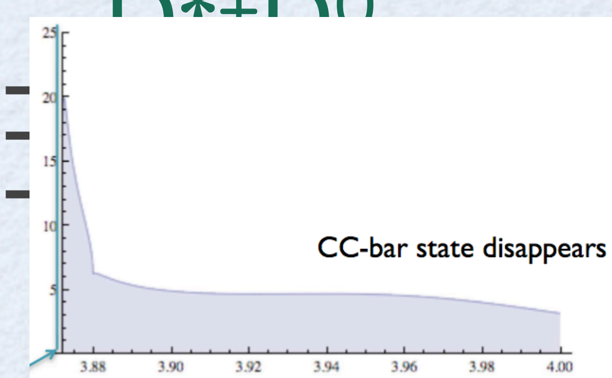
$J/\psi\,\rho$-like

- Case 2

  Large $c\bar{c}$ compo.
  No bound I=1

$J/\psi\,\omega$, $D^{*+}D^{-}$ $\bar{D}^{*0}D^{+}$ $D^{*+}\bar{D}^{0}$

$J/\psi\,\rho$, $D^{*0}\bar{D}^{0}$



$J/\psi\,\omega$-like

$J/\psi\,\rho$ compo: 4%

Energy spectrum
→ Takizawa

Summary

- The $qc\bar{q}\bar{c}+c\bar{c}$ ($J^{PC}=1^{++}$) states are investigated by a quark model.
- ud quark mass diff and ele-mag int btw quarks are introduced → $D^{*0}D^{0}$-$D^{*+}D^{-}$ threshold difference.
- RGM-type calculation with the short range deformation is employed.

Summary

$qc\bar{q}\bar{c}+c\bar{c}$ ($J^{PC}=1^{++}$, ud mass diff & ele)

- X(3872) can be explained as a shallow bound state just below the threshold, with the $c\bar{c}$ component 1%〜40%.
- Large amount of the $c\bar{c}$ component will smare the $c\bar{c}$ peak, which is predicted by the $c\bar{c}$ quark model but seems not to exist.

Summary

$qc\bar{q}\bar{c}+c\bar{c}$ ($J^{PC}=1^{++}$, ud mass diff & ele)

- I=0 is main component for this shallow state, but I=1 component is also mixed by 1/13〜1/4.

  → $Br(X \to J/\psi\,\pi^3)/Br(X \to J/\psi\,\pi^2)$

- There may be a bound states where I=1 is main component. They will probably not be seen because of the broad width.

Outlook

- Baryons with large width may be understood as (meson-baryon + baryon quark core) systems.
- Heavy mesons with small width may be understood as (2-meson-molecule + multiquark state + QQbar core) systems.

C-parity